Physics 31A 1st Midterm (200 pts Max.)
Spring 2005

Show your works with diagrams, explanations, and clear writings. No credit will be given for answers without diagrams, explanations, and clear writings.

1. (a) The radius of the Moon (R_{moon}) is 1.738 x 10^3 km. Assume the Moon is a sphere and determine its volume to two significant figures.

   (b) A solid sphere having a volume of 1.00 m^3 fits tightly in a cubical box. What's the volume contained within the box?

2. (a) A toy car moves along the z-axis in a manner described by the formula $z(t) = C[A + Bt^2]$ where A, B, and C are constants having units of m^2, m^2/s^2, and m^{-1}, respectively. Determine an expression for the instantaneous speed of the car as a function of time. Find the value of the speed at t = 4.0 s. When is the speed zero?

   (b) Given the vector $\vec{r} = (3.00m)\hat{i} + (2.00m)\hat{j} + (4.00m)\hat{k}$, determine its x-, y-, and z-components. Then find $|\vec{r}|$.

3. (a) A vehicle moves along a straight track according to the formula $x(t) = (0.50\ m/s^2)t^2 - (3.0\ m/s)t + 2.5$ m. (i) Make a sketch of the curve (x versus t) showing the point where t = 0, and both points where x = 0. (ii) Show that the curve is a parabola. (iii) Locate the point in time when x(t) has its lowest value, using the idea that the slope of the curve at that point should be zero.

   (b) Two knights with lances are about to joust. Sir John the Slow gallops south at 5.0 km/h, while Sir Peter the Fast races north at 25 km/h. What is the velocity of John with respect to (i.e., as seen by) Peter? At what speed do they close the gap between one another?

4. (a) Making use of the relationship $a = dv/dt = (dv/ds)(ds/dt)$, derive the expression $v^2 = v_0^2 + 2as$.

   (b) A bag of sand dropped by a would-be assassin from the roof of building just misses Tough Tony, a gangster 2-m tall. The missile traverses the height of Tough Tony in 0.20 s, landing with a thud at his feet. How high was the building? Ignore friction.

5. (a) A human being performing a vertical jump generally squats and then springs upward, accelerating with feet touching the Earth through a distance s_a. Once fully extended, the jumper leaves the ground and glides upward, decelerating until the feet are a maximum height s_{max} off the floor. Assuming the jumping acceleration a to be constant, derive an expression for it in terms of s_a and s_{max}. Ignore friction.

   (b) Range S_R is the total horizontal distance that a projectile travels returning to the same height which it was launched. Show that the maximum range occurs at the projection angle at $\theta = 45^o$, and $S_R(\max) = -\dfrac{2v_0^2}{g}$ where v_0 is the initial velocity at angle θ.

6. For an object moving rapidly through the air, the drag force is approximately proportional to the speed squared: $F_d = Kv^2$. Assuming the object was thrown downward at a speed v_0 initially, find an expression for v as a function of the distance fallen, y. Assume v^2-drag is operative.

7. (a) A 100-kg bale of dried hay falls off a truck traveling on a level road at 88.0 km/h. It lands flat on the blacktop and skids 100 m before coming to rest. Assuming a uniform deceleration, compute the coefficient of kinetic friction.

   (b) Someone wants to push a (100-kg) box full of books along the floor by exerting a constant horizontal force of 600 N. Given that the coefficient of static friction is 0.6 and the coefficient of kinetic friction is 0.1, determine the resulting motion of the box.

8. A projectile (of mass m) is to be fired straight up from the surface of a planet (of mass M) having a radius R. Show that the equation for the minimum speed at which the projectile must be launched if it is to rise to a distance r from the center of the planet is given by $v = \sqrt{2GM(1/R - 1/r)}$.

9. A long, thin rod of mass M and length L lies along the y-axis with its center at the origin. (i) Find the gravitational force it exerts on a point-mass ($m_\bullet$) located on the x-axis a distance x_0 away. (ii) If the mass per unit length, call it λ_m, is constant, find the value of the force as $L \to \infty$.