Physics 31A 2nd Test (100pts MAX)
Winter, 2004

Show your work with diagrams, explanations, and clear writing. No credit will be given for answers without diagrams, explanations, and clear writing.

1. A solid cylinder is placed at rest on an inclined plane at a vertical height *h* and allowed to roll down without slipping. Using forces and torques, show that its speed at the bottom is given by $v = 2\sqrt{gh/3}$.

2. (a) A long, uniform rod of length *L* is balanced vertically on one end, which rests on a rough horizontal floor. After a moment the rod begins to fall, rotating around its bottom end that remains where it was without slipping. The rod makes an angle θ with the vertical as it descends. Show that $w = \sqrt{(3g/L)(1-\cos\theta)}$.

   (b) The human thigh bone, the femur, at its narrowest point resembles a hollow cylinder with an outer radius of roughly 1.1 cm and an inner radius of just about half of that. Taking the compressive strength of the bone to be 170 MPa, how much force will be required to rupture it?

3. (a) A uniform solid ball of radius *R* and mass *m* is at rest at a height *h* atop an inclined plane making an angle θ. Write an expression for the linear speed of the sphere at the bottom of the incline assuming it rolls without slipping. Compare that to the speed a hollow sphere of the same mass and size would attain.

   (b) A boy (mass 30 kg) wishes to play on a centrally pivoted seesaw with his dog Irving (mass 10 kg). When the dog sits 3.0 m from the pivot, where must the boy sit if the 6.5-m-long board is to be balanced horizontally?

4. (a) A Texas pipeline carrying natural gas (ρ =0.90 kg/m^3) with a mass-rate of flow of 1.0 kg/s is 35 cm in diameter. Determine the average speed at which the gas is moving along.

   (b) An object of volume *V* floats in water with a volume V_u up above the surface. (i) Write an expression for its average density in terms of the density of water. (ii) Apply this solution to find the average density of a loaded barge with 20% of its volume above the waterline.

5. (a) Consider the frictionless cart in Fig. 1. If the elastic spring constants are k_1 and k_2, respectively, determine the frequency of vibration in terms of these quantities.

   (b) If a uniform rod of cross-sectional area *A* and length L_0 can sustain a maximum stress of σ_R without rupture, show that it stores an amount of elastic energy given by
   in getting to that stress level.