Chapter 25. Reflection and Refraction of Light

25.2 The Ray Model in Geometric Optics

In the simplification model called the ray model or ray approximation, it is assumed that a wave travels through a medium along a straight line in the direction of its rays. A ray for a given wave is a straight line perpendicular to the wave front; geometric models are based on these straight lines. This approximation also neglects diffraction effects.

25.3 The Wave Under Reflection

Specular reflection occurs when the reflecting surface is smooth; reflection from a rough surface is called **diffuse reflection.** In the case of reflection from a smooth surface, the angle of incidence equals the angle of reflection; the angles are measured between the normal to the surface and the respective rays. *The path of a light ray is reversible, an important property in geometric optics.*

Consider the situation in the figure. A light ray is incident obliquely on a smooth, planar surface that forms the boundary between two transparent media of different indices of refraction. A proportion of the ray will be reflected back into the original medium, while the remaining fraction will be transmitted into the second medium. *The incident, reflected, and refracted rays and the normal to the interface are all in the same plane.*

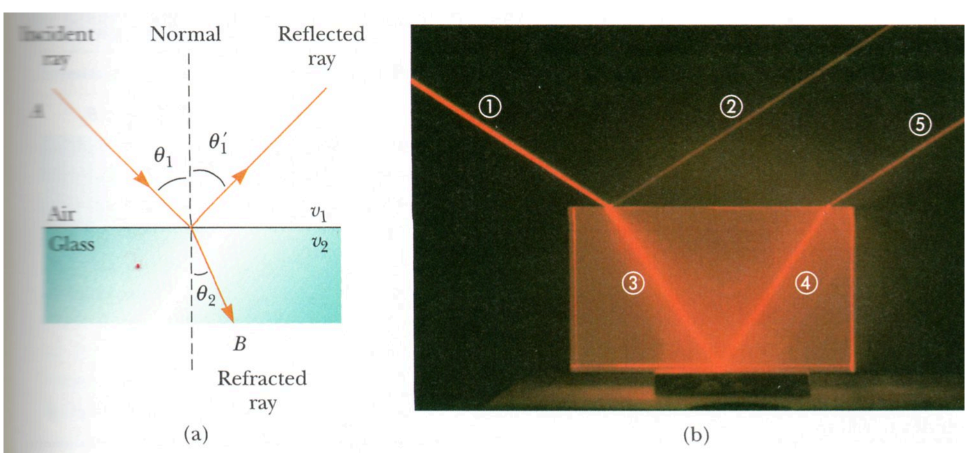


Figure 25.8

The **law of reflection** states the angle of incidence equals the angle of reflection. $\theta_1' = \theta_1$

25.4 The Wave Under Refraction

Reflection refers to the change in direction of a light ray upon striking obliquely the interface between two transparent media. The angle of refraction (between the refracted ray and the normal) depends on the properties of the two media and the angle of incidence as expressed by Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$. The incident ray, reflected ray, and reflected (transmitted) ray like in the same plane. *As a ray travels from one medium to another, the speed of the wave changes but the frequency does not change.*

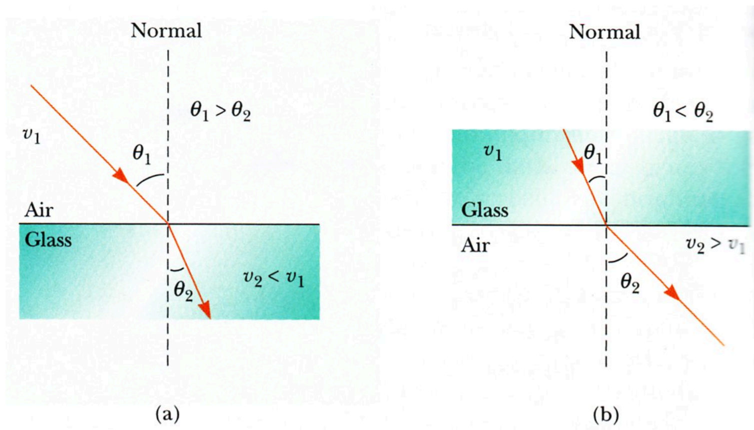


Figure 25.9

Snell's law of refraction can be expressed in terms of the speeds of light in the media on either side of the refracting surface, $\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} =$ constant.

The **index of refraction** of a transparent medium is defined as the ratio of the speed of light in vacuum to the speed of light in the medium. $n \equiv \frac{c}{v}$ (speed of light in vacuum/speed of light in a medium)

The index of refraction of a given medium can be expressed as the ratio of the wavelength of light in vacuum to the wavelength in that medium, $n = \frac{\lambda_0}{\lambda_n}$.

The frequency of a wave is characteristic of the source. Therefore, as light travels from one medium into another of different index of refraction, the frequency remains constant but the wavelength changes.

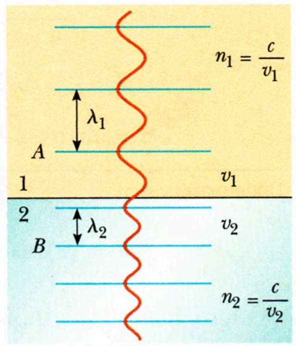


Figure 25.11

25.5 Dispersion and Prisms

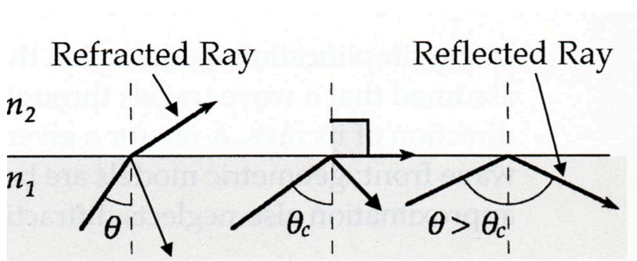
For a given material, the index of refraction is a function of the wavelength of the light passing through the material. (The wavelength depends in turn on the speed). This effect is called dispersion. In particular, when light passes through a prism, a given ray is refracted at two surfaces and emerges bent away from its original direction by an angle of deviation, δ. Due to dispersion, δ is different for different wavelengths.

25.6 Huygens's Principle

Every point on a given wave front can be considered as a point source for a secondary wavelet. At some later time, the new position of the wave front is determined by the surface tangent to the set of secondary wavelets.

25.7 Total Internal Reflection

Total internal reflection, illustrated in **Figure 25.1** is possible only when light rays traveling in one medium are incident on a boundary between the first medium and a second medium of lesser index of refraction. The angle θ_c shown in the figure is called the critical angle.



Total internal reflection of light occurs at angles of incidence $\theta_1 \geq \theta_c$ where $n_1 > n_2$

Figure 25.1

The **critical angle of total internal reflection** is the minimum angle of incidence for which total internal reflection can occur. *Total internal reflection is possible only when a light ray is directed from a medium of higher index of refraction into a medium of lower index of refraction,* $\sin\theta_c = \frac{n_2}{n_1}$ (for $n_1 > n_2$)

Chapter 26. Image Formation by Mirrors and Lenses

26.1 Images Formed by Flat Mirrors

An image formed by a flat mirror has the following properties:

1. The image is as far behind the mirror as the object is the front.
2. The image is actual-size, virtual, and upright. (By upright, we mean that, if the object arrow points upward, so does the image arrow.)
3. The image has an apparent right-left reversal.

The **lateral magnification of a spherical mirror** can be stated either as a ratio of image size to object size or in terms of the ratio of image distance to object distant, $M = \frac{h'}{h} = -\frac{q}{p}$

26.2 Images Formed by Spherical Mirrors

A spherical mirror is a reflecting surface that has the shape of a segment of a sphere. A concave mirror is one that reflects light from the inner concave surface; a convex mirror reflects light from the outer convex surface. When using the mirror equation, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, carefully adhere to the sign conventions stated in the Suggestion, and Skills and Strategies section.

Real images are formed at a point when reflected light following rays in a ray appear to diverge from the image point.

The point of intersection of any two of the following grays in a ray diagram for mirrors locates the image:

1. The first ray is drawn from the top of the object parallel to the principal axis and is reflected back through the focal point, F.
2. The second ray is drawn from the top of the object through the focal point and is reflected parallel to the axis.
3. The third ray is drawn from the top of the object through the center of curvature, C, and is reflected back on itself.
4. The fourth ray is drawn from the top of the object to the center of the mirror's surface and reflects on the other side of the principal axis, with an angle of reflection equal to its angle of incidence.

26.4 Thin Lenses

Images formed by thin lenses can be located and described by pictorial representations called ray diagrams.

To locate the position of an image formed by a converging lens, draw the following three rays from the top of the object:

1. Ray 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through the focal pint on the back side of the lens.
2. Ray 2 is drawn through the center of the lens. This ray continues in a straight line.
3. Ray 3 is drawn through the focal point on the front side of the lens (or as if coming from the focal point if $p < f$) and emerges from the lens parallel to the principal axis.

In the case of a diverging lens, the following three rays are drawn from the top of the object:

1. Ray 1 is drawn parallel to the principal axis, and after refraction emerges directed away from the focal point on the front side of the lens.
2. Ray 2 is drawn through the center of the lens and continues in a straight time.
3. Ray 3 is drawn in the direction toward the focal point on the backside of the lens and emerges from the lens parallel to the principle axis.

Note that any two rays can be used to locate the object; the third ray is useful check on the accuracy of your diagram.

EQUATIONS AND CONCEPTS

The **mirror equation** is used to locate the position of an image formed by reflection of paraxial rays. The focal point of a spherical mirror is located midway between the center of curvature and the center of the mirror.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad (26.6)$$

$$f=\frac{R}{2} \qquad (26.5)$$

The **lateral magnification of a spherical mirror** can be stated either as a ratio of image size to object size or in terms of the ratio of image distance to object distance.

$$M = \frac{h'}{h} = -\frac{q}{p} \qquad (26.2)$$

A magnified image of an object can be formed by a **single spherical refracting surface** of radius R, which separates two media whose indices of refraction are n_1 and n_2.

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \qquad (26.8)$$

$$M = -\frac{n_1 q}{n_2 p} \qquad (26.9)$$

A special case is that of the virtual image formed by a **planar refracting surface** (R= ∞).

$$\frac{n_1}{p} = -\frac{n_2}{q} \qquad q = -\frac{n_2}{n_1}p \qquad (26.10)$$

The **focal length of a thin lens** is determined by the characteristic properties of the lens (index of refraction n and radii of curvature R_1 and R_2).

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (26.13)$$

The **lateral magnification**, M, will be negative when the image is inverted.

$$M = \frac{h'}{h} = -\frac{q}{p} \qquad (26.11)$$

The **thin lens equation** is identical to the mirror equation and can be used with both converging (positive f) and diverging (negative f) lenses.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad (26.12)$$

SUGGESTIONS, SKILLS, AND STRATEGIES

A major potion of this chapter is devoted to the development and presentation of equations, which can be used to determine the location and nature of images formed by various optical components acting either singly or in combination. It is essential that these equations be used with the correct algebraic sign associated with each quantity involved. You must understand clearly the sign conventions for mirrors, refracting surfaces, and lenses. The following discussion represents a review of these sign conventions.

Sign Conventions for Mirrors

Equations $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R}$ $\quad M=\frac{h'}{h}=-\frac{q}{p}$

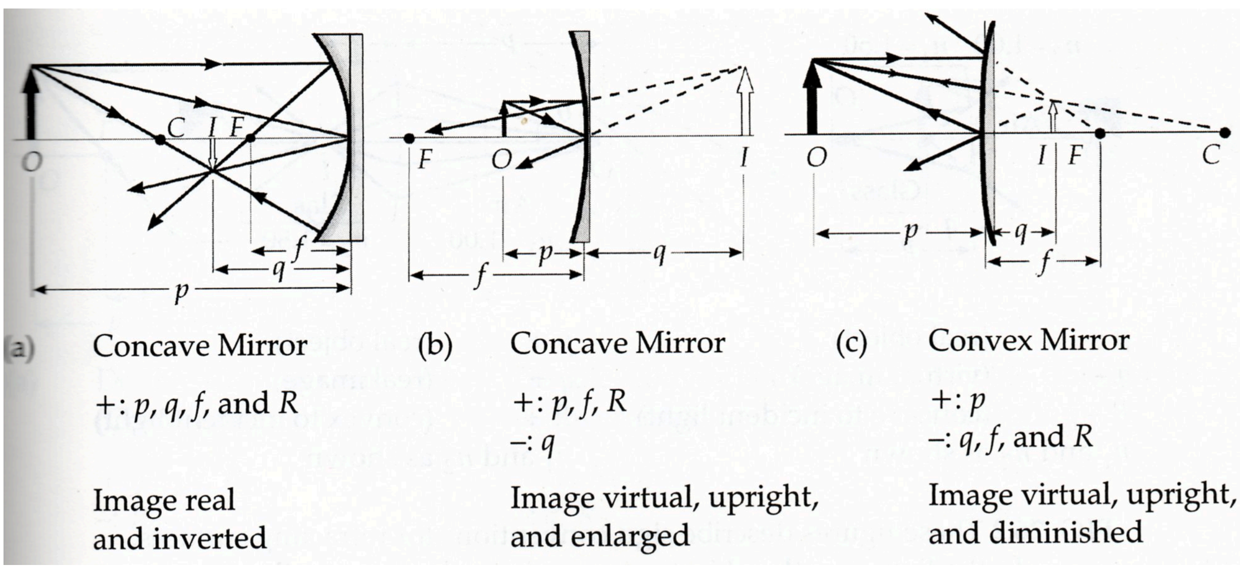
The front side of the mirror is the region on which light rays are incident and reflected.

p is + if the object is in front of the mirror (real object).
q is – if the object is in back of the mirror (virtual object).

q is + if the image is in front of the mirror (real image).
q is – if the image is in back of the mirror (virtual image).

Both f and R are + if the center of curvature is in front of the mirror (concave mirror).
Both f and R are – if the center of curvature is in back of the mirror (convex mirror).

If M is positive, the image is upright.
If M is negative, the image is inverted.



Figures describing sign conventions for mirrors.

Sign Conventions for Refracting Surfaces

Equations: $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ $\quad M=\frac{h'}{h}=-\frac{n_1 q}{n_2 p}$

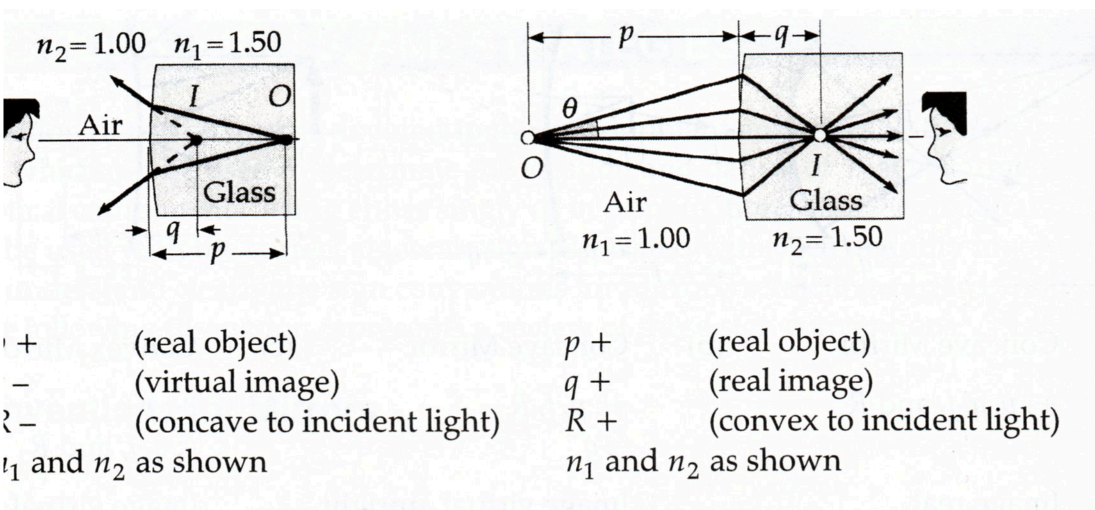
In the following table, the front side of the surface is the side from which the light is incident.

p is + if the object is in front of the surface (real object).
p is – if the object is in back of the surface (virtual object).

q is + if the image is in back of the surface (real image).
q is – if the image is in front of the surface (virtual image).

R is + if the center of curvature is in back of the surface.
R is – if the center of curvature is in front of the surface.

n_1 refers to the index of refraction of the medium on the side of the interface from which the light comes.
n_2 refers to the index of refraction of the medium into which the light is transmitted after refraction at the interface.



These figures describe sign conventions for refracting surfaces.
In the first case, the object appears closer than it actually is.
In the second case, an object beyond the glass appears to be in the glass.

Sign Conventions for Thin Lenses

Equations: $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2})$ $\qquad M = \frac{h'}{h} = -\frac{q}{p}$

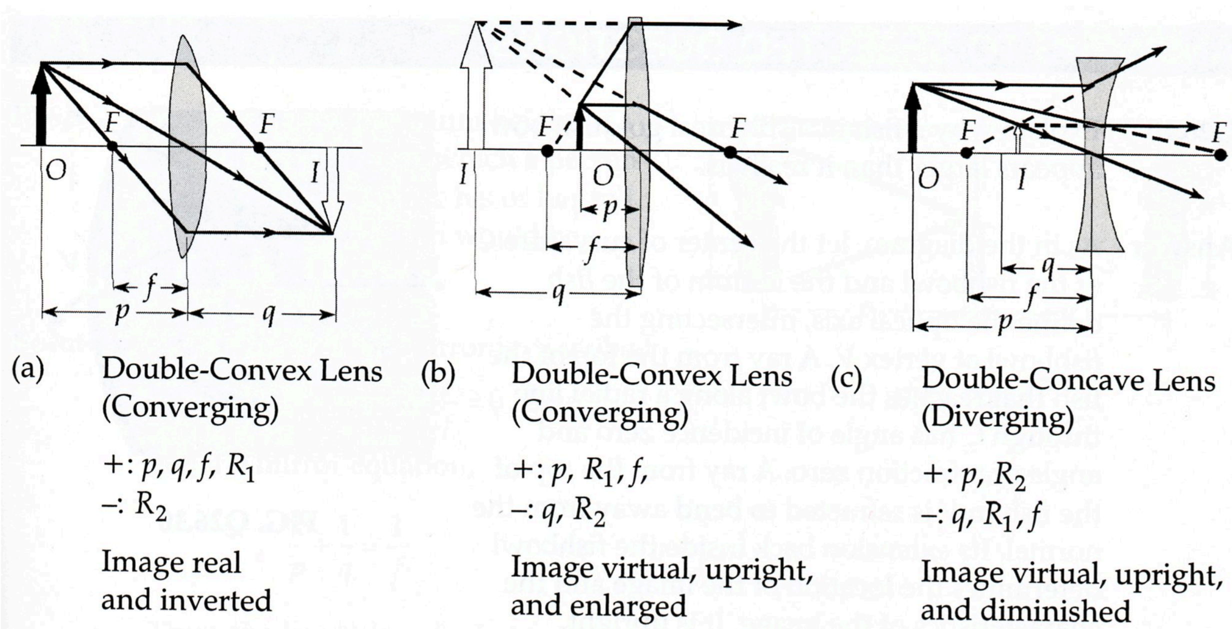
In the following table, the front of the lens is the side from which the light is incident.

p is + if the object is in front of the lens.
p is – if the object is in back of the lens.

q is + if the image is in back of the lens.
q is – if the image is in front of the lens.

f is + if the lens is thickest at the center.
f is – if the lens is thickest at the edges.

R_1 and R_2 are + if the center of curvature is in back of the lens.
R_1 and R_2 are – if the center of curvature is in front of the lens.



Figures describing the sign conventions for various thin lenses.

Chapter 27. Wave Optics

27.1 Conditions for Interference

In order to observe **sustained interference** in light waves, the following conditions must be met:

- The sources must be **coherent**; they must maintain a **constant phase** with respect to each other.
- The sources must be **monochromatic** – of a single wavelength.

27.2 Young's Double-Slit Experiment

A schematic diagram illustrating the geometry used in Young's double-slit experiment is shown in Figure 27.1 below. The two slits S_1 and S_2 serve as coherent monochromatic sources. When L (the distace from source plane to viewing screen) is much greater than d (the distance etween sources), the **path difference** $\delta = r_2 - r_1 = d\sin\theta$.

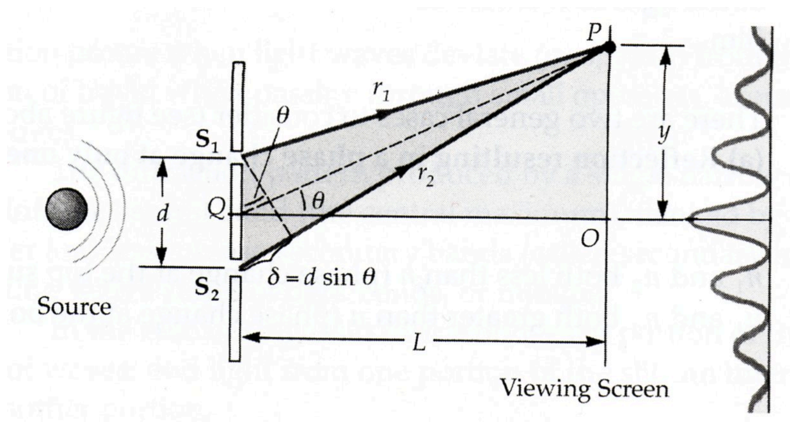
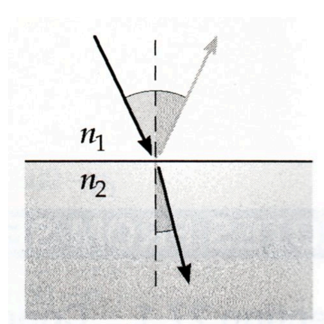


Figure 27.1 Young's double slit experiment. The relative intensity of light on the screen is shown on the right.

27.4 Change of Phase Due to Reflection

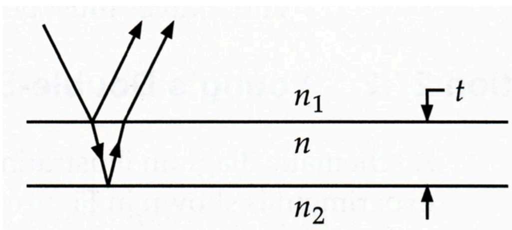
Consider a light wave traveling in a medium with an index of refraction n_1. When partial reflection occurs at the surface of a medium with an index of refraction n_2:



- If $n_1 < n_2$, the reflected ray experiences a phase change of $180°$.
- If $n_1 > n_2$, there is no phase change in the reflected ray.
- There is no phase change in the transmitted ray regardless of the relative values of n_1 and n_2.

27.5 Interference in Thin Films

In order to predict constructive or destructive interference in thin films you must consider:



- The difference in path-length traveled by the two interfering waves;
- Any expected changes in phase due to reflection;
- The change in wave length of the light as it enters th film

There are two general cases to consider (see figure above):

(a) Reflection resulting in a phase change at only one surface of the film.

n_1 and n_2 both less than n (phase change at the top surface)

n_1 and n_2 both greater than n (phase change at the bottom surface)

Constructive interference will occur under these conditions, when the path difference (which equals 2t) is an odd number of half wavelentgths, $2t = (2m+1)\frac{\lambda_n}{2}$.

Thus, the film thickness is $t = (\frac{m+1}{2})\frac{\lambda_n}{2}$. Here, $\lambda_n = \frac{\lambda}{n}$ *is the wavelength as measured in the film (the wavelength in vacuum, divided by the index of refraction of the film).* The condition for constructive interference in this case then becomes $t = (\frac{m+1}{2})\frac{\lambda}{2n}$.

Destructive interference will occur under these conditions, when the path difference, 2t, equals an integer number of wavelengths so that $t = \frac{m\lambda}{2n}$.

(b) Reflection resulting in phase changes at both top and bottom surfaces of the film (or at neither surface).

$n_1 < n$ and $n_2 > n$ (phase change at both surfaces)
$n_1 > n$ and $n_2 < n$ (no phase change at either surface)

In this case, these two phases changes are offsetting and interference of the reflected rays depends only on the difference in distance traveled by the two reflected rays and the index of refraction of the film.

Constructive interference will occur when the path difference equals an integer number of wavelengths; the film thickness must be an integer number of half wavelengths, the film thickness must be an integer number of half wavelengths, $t = \frac{m\lambda}{2n}$.

Destructive interference in this case will be observed when the path difference equals an odd number of half wavelengths; that is when $t = (m + \frac{1}{2})\frac{\lambda}{2n}$.

27.6 Diffraction Patterns

Diffraction occurs when light waves deviate (or spread) from their initial direction of travel when passing through small openings, around obstacles, or by sharp edges.

The diffraction pattern produced by a single narrow slit consists of a broad, intense central band (the central maximum), flanked by a series of narrower and less intense secondary bands (called secondary maxima) alternating with a series of dark bands, or minima.

In the case of single slit diffraction, each portion of the slit acts as a source of waves; and light from one portion of the slit can interfere with light from another portion.

The resultant intensity on the screen depends on the angle θ between the perpendicular to the plane of the slit and the direction to a point on the screen.

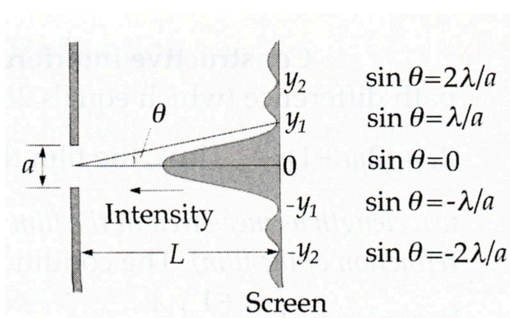


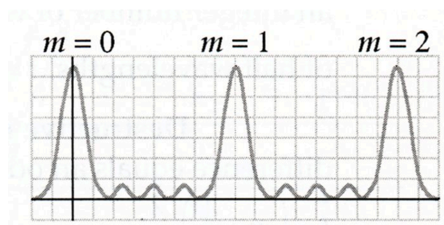
Fig. 27.3 Single Slit Diffraction Pattern

One type of diffraction, called Fraunhofer diffraction, occurs when the rays reaching the observing screen are approximately parallel.

27.7 Resolution of Single-Slit and Circular Apertures

When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved. This limiting condition of resolution is known as **Rayleigh's criterion.**

27.8 The Diffraction Gratting



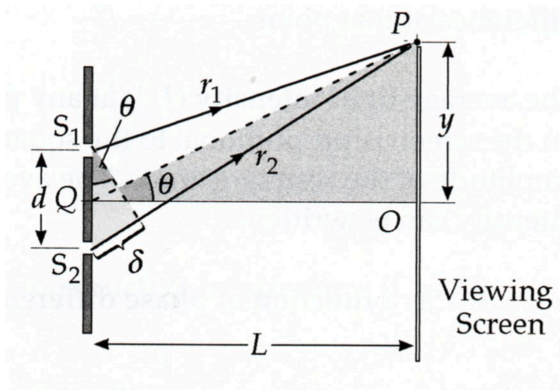
A diffraction grating, consisting of many equally spaced parallel slits, separated by a distance *d*, will produce an diffraction pattern. There will be a series of principle maxima (bright lines) for each wavelength component in the incident light. *The figure illustrates the case for which the incident light contains a single wavelength component.* Although not shown in the figure above, there will also be maxima corresponding to *m*=-1, -2, … These maxima occur at angles θ, measured from the line perpendicular to the grating, where $m\lambda = d\sin\theta$.

If the incident light contains a second wavelength component, a second series of principle maxima (in general with a different intensity) will be present in the diffraction pattern. In a given spectral order, denoted by the number *m*, there will be principle maxima corresponding to each wavelength component incident on the grating.

EQUATIONS AND CONCEPTS

In **Young's double-slit experiment**, two slits, S_1 and S_2, separated by a distance *d*, serve as monochromatic coherent sources. The light intensity at any point on the screen is the resultant of light reaching the screen from both slits. *As illustrated in the figure, a point P on the screen can be identified by the angle* θ *or vy the distance u from the enter of the screen.* When using $y = L\tan\theta$, *y* is measured from the center of the interference pattern and *L* is the distance from the double slit to the screen. Light from the two slits reaching any point on the screen (except the center) travels unequal path lengths. This difference in length of path, δ, is called the path difference.

$\delta = r_2 - r_1 = d\sin\theta$
$y = L\tan\theta$



Constructive interference (bright fringes) will appear at points on the screen for which the path difference is equal to an integral multiple of the wavelength. The positions of bright fringes can also be located by calculating their distance from the center of the screen (y). In each case, the number m is called the order number of the fringe. *The central bright fringe* ($\theta = 0,\ m = 0$) is *called the zeroth-order maximum.*

$\delta = d\sin\theta_{bright} = m\lambda$ for m=0, ±1, ±2, …

$y_{bright} = L\tan\theta_{bright}$

$y_{bright} \approx m\frac{\lambda L}{d}$ for small θ

Destructive interference (dark fringes) will appear at points on the screen, which correspond to path differences of an odd multiple of half wavelengths. For these points of destructive interference, waves which leave the two slits in phase arrive a the screen $180°$ out of phase.

$\delta = d\sin\theta_{bright} = (m + \frac{1}{2})$ for m=0, ±1, ±2, …

$y_{dark} = L\tan\theta_{dark}$

$y_{dark} \approx (m + \frac{1}{2})\frac{\lambda L}{d}$ for small θ

The **phase difference** ϕ between the two waves at any point on the screen depends on the path difference at that point.

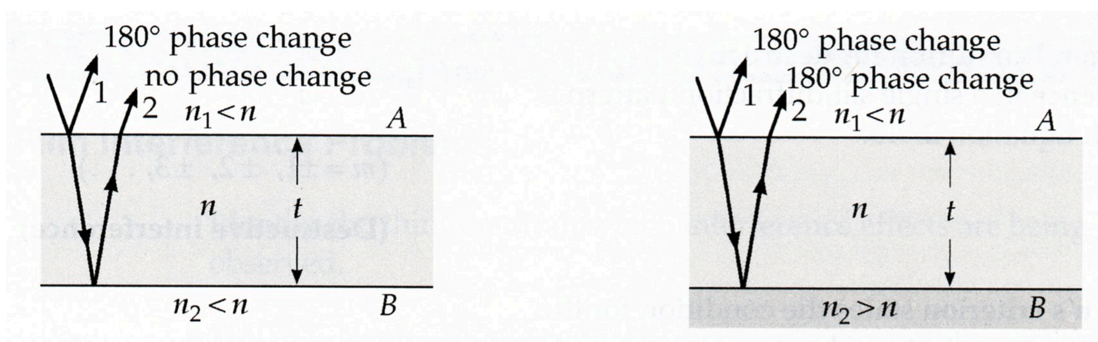
$\phi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda}d\sin\theta$

The **average light intensity** (I_{av}) at any point *P* on the screen is proportional to the square of the amplitude of the resultant wave. The average intensity can be written:

- As a **function of phase difference** ϕ**,** $I_{av} = I_{max}\cos^2(\frac{\phi}{2})$
- As a **function of the angle (**θ**)** subtended by the screen point at the source midpoint; or $I_{av} = I_{max}\cos^2(\frac{\pi d\sin\theta}{\lambda})$
- As a **function of the distance** (y) from the center of the screen, $I_{av} \approx I_{max}\cos^2(\frac{\pi d}{\lambda L}y)$ for small θ

The wavelength of light in a medium having an index of refraction *n* is smaller than the wavelength in vacuum. In thin-film interference the wavelength of light within the film is $\lambda_n = \frac{\lambda}{n}$.

Interference in thin film depends, on wavelength, film thickness, and the indices of refraction of the film and surrounding media. *Differences in phase may be due to path difference or phase change upon reflection*. There are two general cases.



<u>Case (1)</u>

Phase change at only one film surface

(n_1<*n* and n_2<*n*) or (n_1>*n* and n_2>*n*). Indices of refraction of media on both sides of the film are less than that of the film (as shown in figure above left) or both greater than that of the film.

Constructive interference (Case 1)

$$2nt = (m + \frac{1}{2})\lambda$$

(m=0, 1, 2, …)

Destructive interference (Case 1)

$2nt = m\lambda$

(m=0, 1, 2, …)

Case (2)

Phase changes at either both surfaces or at neither surface (n_1<n<n_2) or (n_1>n>n_2). Film is between two media either of which has an index of refraction greater than that of the film and the other a smaller index (as shown figure above right).

Constructive interference (Case 2)

$2nt = m\lambda$

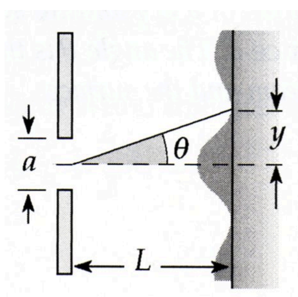
(m=0, 1, 2, …)

Destructive interference (Case 1)

$2nt = (m + \frac{1}{2})\lambda$

(m=0, 1, 2, …)

A single-slit diffraction pattern consists of a central maximum with a series of alternating bright and dark bands (minima) on each side. The intensity at a given point on the screen depends on the angle between the direction to the central maximum and the direction to that point.



The **general condition for destructive interference** in a single slit diffraction pattern is sated in $\sin\theta_{dark} = m\frac{\lambda}{a}$ (m=0, ±1, ±2, …) – Destructive interference

Rayleigh's criterion states the condition for the resolution of two images due to nearby sources. For a **slit,** the angular separation between the sources must be greater than the ratio of the wavelength to slit width if the two sources are to be resolved.

$\theta_{min} = \frac{\lambda}{a}$ **(Limiting angle of resolution for a slit)**

For a **circular aperture**, the minimum angular separation depends on *D*, the diameter of the aperture (or lens).

$\theta_{min} = 1.22\frac{\lambda}{a}$ **(Limiting angle of resolution for a circular aperture)**

A **diffraction grating** (an array of a large number of parallel slits separated by a distance *d*) will produce an interference pattern in which there is a series of maxima for each wavelength. *Maxima due to wavelengths of different values comprise a spectral order denoted by order number m.*

$d\sin\theta_{bright} = m\lambda$ (*m*=*0, 1, 2, 3 …*) (27.16)

Each spectral order will contain a line characteristic of each wavelength. Equation 27.16 gives the angle of deviation for constructive interference (bright lines in the spectrum).

Bragg's law gives the conditions for **constructive interference** of x-rays reflected from the parallel planes of a crystalline solid separated by a distance *d*. *The angle* θ *is the angle between the incident beam and the surface.*

$2d\sin\theta = m\lambda$ (*m*= *1, 2, 3 …*) (27.17)

SUGGESTIONS, SKILLS, AND STRATEGIES

Thin-Film Interference Problems

- Identify the thin film from which interference effects are being observed.
- The type of interference that occurs in a specific problem is determined by the phase relationship between that portion of the wave reflected at the upper surface of the film and that portion reflected at the lower surface of the film.
- Phase differences between the two portions of the wave occur because of differences in the distances traveled by the two portions and by phase changes occurring upon reflection.

Phase Difference due to Path Difference

The wave reflected from the lower surface of the film has to travel a distance equal to twice the thickness of the film before it returns to the upper surface of the film where it interferes with that portion of the wave reflected at the upper surface.

Phase Change due to Reflection

When a wave traveling in a particular medium reflects off a surface having a higher index or refraction than the one it is in, a $180°$ phase shift occurs. This has the same effect as if the wave had traveled a lesser distance of $\frac{1}{2}\lambda$. This effect must be considered in addition to the phase difference related to the greater distance traveled by one of the waves.

When distance and phase changes upon reflection are both taken into account:

Constructive interference will occur when the effective path difference is an integral multiple of λ-zero, λ, 2λ, 3λ, …

Destructive interference will occur when the effective path difference is an odd number of half wavelengths - $\frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda, \ldots$